Walking out graphs

by Ji Shen

Ultrasound motion detectors and associated computer software have been widely used to help students learn concepts related to motion and graphing (Brasell 1987; Mokros and Tinker 1987). Typically, a motion detector can detect the position of an object within its range of ultrasound signals. By sending these signals at a constant frequency, the detector can log position data over time. Associated software provides a rich set of representations and spontaneous calculations (e.g., velocity, acceleration). However, multiple representations do not necessarily lead to scientific understanding. They may confuse students. Without practicing the transformations among multiple representations, students may not develop the scientific habit of mind of systematically organizing their knowledge. In this Walking Out Graphs lesson, students experience several types of representations used to describe motion, including words, sentences, equations, graphs, data tables, and actions. The most important theme of this lesson is that students have to understand the consistency among these representations and form the habit of transforming among these representations (Shen and Confrey 2007).

The idea for this lesson came from a project I developed with a colleague for an after school program to teach graphing to middle school students. Students were asked to analyze the given data collected from motion sensors in a hypothetical car accident at an intersection. Students needed to make sense of the data using graphs, and draw a conclusion on who was at fault in the accident. What’s described here is only the core lesson on graphing motion. The reader should keep in mind that it will be more meaningful for students if it’s embedded in a similar real-life context, such as a car accident.

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From observations of my personal teaching, students appreciate the activities and the opportunity to engage in intellectual work. The lesson can be adapted (content, format, and sequence) to fit the levels of the participants. As it is presented here, students would need to have some basic algebra knowledge (simple linear function) and have acquired basic knowledge about motion (relationships among displacement, velocity, and time).
Students’ initial ideas

Students usually bring many ideas with them to the classroom (Linn 2006), some of which may be misconceptions or disconnected. For instance, students can hardly distinguish between the concepts of velocity and acceleration. To uncover the prior knowledge students hold, begin the lesson by eliciting their verbal descriptions of motion. You can simply pose the following question to the class: “What words do you use to describe motion (of an athlete, a ball, an animal, etc.)?” Students may generate a wide range of words covering both kinematics (e.g., movement, position, speed, circular, fast) and dynamics (e.g., gravity, mass, weight). These words are often used in the context of everyday conversation, which may make it difficult for students to use them in a coherent and scientific manner. For instance, the term fast may be used to describe how fast an object changes its position or how fast it changes its speed. Surprisingly, one of the most important variables, time, is often left out. Highlight for students that describing motion as it is (kinematics) is the focus of this lesson. Students will study why things move in such a way (dynamics) after they have learned how things move. Drawing upon everyday use of these terms and later distinguishing these terms in a scientific way may help students identify the sources of their misconceptions (e.g., people use the words mass and weight interchangeably in everyday conversation when saying that one weighs 150 pounds, which is a unit of mass).

Organizing ideas into sentences

In this step you may ask students to provide full sentences to describe the relationships among displacement, velocity, and time. You may ask students to write down their individual sentences and then share with the whole class what they write. Alternatively, you may ask a volunteer to share with the whole class his/her sentence and then ask other students to add on or critique the initial sentence. A target response is that velocity is the rate of change of displacement over time.
Students usually use scalars such as speed and distance instead of vectors. Since direction is either positive or negative in a one-dimensional world, the vectors of displacement and velocities can be relatively easily explained. Using students’ sentences, the instructor may draw links among these ideas in a concept map. A sample concept map is illustrated in Figure 1.

Symbols and equations
To simplify the representation of the relationships among variables used to describe motion, standard symbols can be introduced (v as velocity, d as distance, t as time, etc.). Students are asked to represent in formulas the sentences they have written. (e.g., \( \Delta d = v \Delta t \)). They also need to reiterate these formulas in sentences using their own words and discuss why these mathematical representations help explain motion.

Data table
The next step is to ask a volunteer student to walk straight from one side of the classroom to the other. The student needs to define the initial point (either a door, the other side of the classroom, or another point) and mark on the ground some distance tick marks. These procedures set up the coordinates of the student’s motion, which are critical when the student communicates with others his/her motion. While the student is walking, the instructor counts from 1 to 10 seconds and puts the position (distance from origin) data on a table on the board (see Figure 2 for an example).

Mapping the data table onto a graph
The data table created on the board by the instructor can be directly converted into a position-time graph (see line A in Figure 3). A position-time graph for one-dimensional motion has two components: the horizontal axis represents time; the vertical axis represents distance from the origin. Students then mark each data point (a particular distance at a given time) on the coordinates. They need to understand that each data point represents a pair of numbers with physical meanings: one representing time and the other representing distance.

Students may naturally connect the data points and draw a straight line. Ask students why they need a line in addition to the data points. For instance, a line can extend the information of a set of data points in terms of extrapolation and interpolation. To illustrate the meaning

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**FIGURE 4** Student instructions for walking out graphs

This activity takes approximately one hour for walking out each graph. A typical classroom space may only fit two groups. So a hallway (or other open space) is needed for more groups. Each group must answer the following three guiding questions.

**Guiding questions**

1. Discuss with your partners and write down how you plan to walk out the following lines on the position-time and velocity-time graphs.
2. Try them out. Explain how you walked.
3. Convert the lines on the position-time graph to the velocity-time graph and vice versa.
of the line, ask students to explain how to draw a line if one travels at a slower (or faster) rate (see line B in Figure 3). A more interactive version is to have a pair of students come up and walk out these two lines (e.g., lines A and B in Figure 3) while the instructor is counting. After the walking, ask the students to explain why they walked the way they did. Students will probably bring up the term “slope” when explaining the lines they draw. Otherwise the instructor may bring this up: By comparing the slopes of two lines, say line A and line B (see Figure 3), one may determine who walks faster. Some students may explain that at a certain time, A has covered a greater distance. Others may also explain that at a certain distance, B uses more time. These alternative explanations are consistent and lead to the concept of (average) velocity, which is the distance traveled over a time interval.

By introducing the general form of a line \( y = mx + b \), the instructor ties this step back to what students have learned in algebra class about linear equations. Ask students to link this equation to the lines: What do the symbols \( m \), \( b \), \( x \), and \( y \) in the equation refer to in the graph? Then ask students to connect the mathematical representation with their physics content: What do these symbols mean in physics? What kind of motion does the equation or the line represent? Through discussion and explanation, students need to understand that both a straight line in the position-time graph and a linear equation represent constant velocity motion. The constant \( m \) in front of \( x \) in the equation is the slope for the line, referring to the velocity. For instance, line A in Figure 3 has a greater slope than line B, meaning that line A represents a faster constant velocity motion.

Similarly, a velocity-time graph is introduced using the methods discussed above. A velocity-time graph for one-dimensional motion has two components: the horizontal axis represents time; the vertical axis represents velocity. For middle school students, this is a very challenging graph. The instructor may only focus on the conceptual level and the general shape of the velocity-time graphs.

**Small-group performance assessment**

Next, students further strengthen their understanding by completing formative performance assessments. Students break into small groups of three to four students and discuss how to walk out a set of lines on both the position-time and velocity-time graphs. Walking out each graph may take approximately an hour to accomplish (including class discussions). Depending on the available instructional time, the instructor may decide to reduce the number of lines that students need to walk out.

To increase the contrast, the instructor may create two sets of lines with the same shapes (see Figure 4). Students need to discuss and plan their walk before they carry out the real walk. Motion detectors can certainly help students record data and generate the graphs, which provide instant feedbacks to their performance.

More importantly, students need to transform the position-time graph to a velocity-time graph and the velocity-time graph to an position-time graph. For instance, a straight line on the position-time graph corresponds to a flat straight line on the velocity-time graph; but a straight line on the velocity-time graph corresponds to a parabola on the position-time graph. Students are expected to understand that the two sets of graphs look the same but are distinctively different in terms of physical meanings. By walking out the graphs, students have the opportunity to physically tie these graphs to their own motions. This provides a concrete experience for students to distinguish the two types of graphs. Students may also draw challenging lines (e.g., a zigzag line on the velocity-time graph) for their peers to walk out.

**Class discussion: Tying everything together**

Use a whole-class discussion to summarize what students have done. Ask students to draw the lines they walked out on the board, and to explain why they drew the lines and how they walked out the lines. The discussion may focus on what difficulties students faced (typically, the velocity-time graph is very challenging for students to walk out), what predictions they found incorrect (e.g., a negative slope on the velocity-time graph refers to walking in the opposite direction; see parts of lines C and D on the velocity-time graph in Figure 4), and what understandings they gained. A detailed explanation of each line (explained by students), and the conversion between the two graphs is extremely important. Through this discussion and reflection, students entertain the physics meanings of slope (flat versus slanted, small slope versus big slope, and directions of slopes), intercept, and shape on the distance-time and velocity-time graphs.

Ask students to interpret the transitional points on some of the lines. For example, lines C and D in the distance-time graph in Figure 4 are non-smooth curves. During actual walking, one has to take some time to stop (line C) or start moving (line D, the second turning
point), as opposed to the ideal situation presented in the graph. Comparing and contrasting the ideal shapes of the curves and real walking data further deepen students’ understanding of processes of motion.

Conclusion

Overall, this is an interesting, engaging, and flexible activity that can easily be tied to everyday experience (e.g., movements of athletes in sports games). It not only asks students to think about physics concepts (e.g., velocity) and mathematics (e.g., algebra, graphing), but also helps students develop their process skills such as planning, explaining, collecting data, experimenting (walking), and transforming. From my experience, it is not solely the multiple representations, but the transformations among them, that help students find inconsistencies, seek scientific understanding, generate creative thoughts, and tie school knowledge back to their experiences.

References


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